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A continuous Wick rotation for spinor fields and supersymmetry in Euclidean space

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Abstract: We obtain a continuous Wick rotation for Dirac, Majorana and Weyl spinors $\psi \to \exp(\frac{1}{2}\theta\gamma^4\gamma^5)\psi$ which interpolates between Minkowski and Euclidean field theories.

In quantum field theory , the "Wick rotation" usually denotes a rotation of k_0 in Greens functions. For bosonic fields, such as the electromagnetic field, one may also define a Wick rotation on the fields themselves. Recently, we have found a consistent way of defining a Wick rotation on fermionic fields. The results will be published in Physics Letters B (see also hep-th/9608174) where a complete set of references can be found. Another article treating the Wick rotation for fermionic fields in a canonical formalism is in preparation.

1 Previous approaches.

If one wants to study supersymmetric theories with instantons, one obviously needs a Euclidean field theory for spinors. Also for the study of Donaldson invariants for compact Euclidean manifolds , one must twist the bosonic fields of a $N = 2$ supersymmetric model such that after twisting they become half-integer spin fields. A clear understanding of the relation between Minkowski and Euclidean supersymmetry might help to solve puzzles about reality issues. (Do twisting and Wick rotation commute?) Similarly, the recent interest in magnetic monopoles and black holes leads one to ask questions about the Euclidean formulation of such theories.

There are also non-supersymmetric reasons for being interested in Euclidean field theories for spinors, and their relation to Minkowski theories. For example, in Fujikawa's approach to anomalies, one regulates the Jacobian in the path integral with

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operators such as $\not{D}\not{D}$. Only in Euclidean space $\not{D}$ is hermitean, and for that reason some authors argue that this scheme only makes sense in Euclidean space. (However, starting in Minkowski spacetime with nonhermitean $\not{D}\not{D}$, one still can obtain momentum integrals which become convergent after the usual analytic continuation in k_0). Further applications of Euclidean field theory are in high-temperature field theory (where the transition from the real time formalism to the imaginary time formalism involves a Wick rotation), and in more formal aspects of field theory (for example to justify in a more convincing way than just to add "$i\epsilon$" to the time that Green's functions are vacuum expectation values, or in the study of convergence of the perturbation series and the effects of renormalons).

Despite these important fundamental issues and applications, relatively little work has been done on the Wick rotation for spinors. If one looks at the lists of contents in recent text books on quantum field theory at an advanced level, one finds at best a discussion of the Wick rotation of the momentum variable k_0 in Green's functions, or a discussion of path integrals for bosons in Euclidean space, but hardly any mention is made of the role fermions must play in the Wick rotation. Instead, most work has concentrated on directly constructing a field theory in Euclidean space whose Green's functions reproduce the analytically continued Green's functions of the Minkowski theory. Here (at least) four approaches have emerged:

(i) The Schwinger approach in which one constructs a **hermitean** action. This precludes $N = 1$ supersymmetry in $d = 4$ Euclidean space because there are no Majorana spinors in Euclidean space.[2] For the $N = 2$ model with Dirac spinors, Zumino constructed a hermitean supersymmetric action, but one of the scalar fields has the wrong sign in front of its kinetic term. Some of the compact symmetries become noncompact in the Euclidean theory, and vice-versa.

(ii) the Osterwalder-Schrader (OS) approach. Here hermiticity in Euclidean space is abandoned (which is not a problem since hermiticity is primarily needed for unitarity, and unitarity only makes sense in a theory with time). The extension of the OS formalism to Majorana spinors and supersymmetry was made by Nicolai. The basic idea of the OS approach for Dirac spinors is to view $\bar{\psi}$ and ψ in Euclidean space as two **independent** complex spinors, while for Majorana spinors in a $N = 1$ Minkowski theory, one writes the action in Euclidean space as $(\psi^T C)\not{\partial}\psi$, just as in the Minkowski case, but one considers ψ as complex (so no longer subject to a reality constraint). Since for susy one only needs properties of spinor bilinears under transposition or the Fierz recoupling formula

[2] A Majorana spinor is a spinor whose Majorana conjugate $\psi^T C$ is proportional to its Dirac conjugate $\bar{\psi}_D$. In Minkowski spacetime, $\bar{\psi}_D = \psi^\dagger i\gamma^0$ but in Euclidean space $\bar{\psi}_D = \psi^\dagger$ (note that the definition $\bar{\psi}_D = -\psi^\dagger\gamma^5$ in Euclidean space is also incompatible with the Majorana condition).

for spinors, but not the reality condition on spinors, $N = 1$ susy in Euclidean space is obtained (but hermiticity is lost).

(iii) the Fubini-Hanson-Jackiw approach, in which one views the radius in Euclidean space as the time coordinate ("radial quantization"). (Note that in string theory the map from the cylinder to the plane $\exp(i\sigma + it) \to \exp(i\sigma + \tau) = z$ does not correspond to radial quantization. Rather, it is $\exp \tau$ which plays the role of radius).

By far the most used approach is that of OS (and its supersymmetric extension by Nicolai). However, all these approaches are either formulated in Minkowski spacetime or in Euclidean space, but no continuous interpolation existed until the work of Mehta.

(iv) in the approach of Mehta, an external metric $g_{\mu\nu}(\theta)$ is introduced, which depends on an angle θ ($0 \le \theta \le \pi/2$), such that for $\theta = 0$ one finds a Minkowski theory for spinors, while at $\theta = \pi/2$ one obtains a Euclidean theory. He also rotates the Dirac matrices appearing in $\not{\partial}$ (by a nonunitary transformation) such that γ^0 and γ^5 become interchanged. The γ^4 (where $\gamma^4 \equiv i\gamma^0$) in $\psi^\dagger i\gamma^0$ is then reinterpreted as γ_E^5. An appropriate choice of the matrix $g_{\mu\nu}(\theta)$ yields the hermitean and $SO(4)$ invariant action $\psi^\dagger \gamma_E^5 (\gamma_E^4 \partial_4 + \vec{\gamma}_E \cdot \vec{\partial} + m)\psi$.

We shall further comment on some of these approaches in the final section, but first we present our approach.

2 The new Wick rotation.

We shall construct a continuous Wick rotation depending on an angle θ ($0 \le \theta \le \pi/2$) for the basic fermionic **fields**. No extraneous metric is introduced, and we do not rotate Dirac matrices or any other constants; rather, as a result of rotating the spinor fields, one can flip the rotation matrices from acting on the spinors to acting on the Dirac matrices, and this will **induce** a unitary rotation of the Dirac matrices. In fact, this rotation will turn out to be a 5-dimensional Lorentz rotation, in the plane spanned by the Minkowski and Euclidean time coordinates. The basic problem is to determine how the spinor fields transform under a Wick rotation; once that problem is solved, all results should follow and no further input should be necessary.

Of course, the Minkowski time coordinate t rotates into the Euclidean "time" coordinate τ according to

$$t \to e^{-i\theta} t_\theta \ ; \ t_{\theta=0} = t, t_{\theta=\pi/2} = \tau \tag{1}$$

At all θ, t_θ is real, hence the arrow indicates a **substitution** , not an equality.

In addition to transforming the time coordinate, we know already from the example of the electromagnetic field A_μ that one should also transform fields. Namely, the time component A_0 transforms in a way similar to t, namely[3]

$$A_0(\vec{x}, t) \to e^{-i\theta} A_0^\theta(\vec{x}, t_\theta) \tag{2}$$

At $t = 0$, $A_0^{\theta=0}(\vec{x}, t)$ equals the Minkowski field $A_0(\vec{x}, t)$ while at $\theta = \pi/2$, $A_0^{\theta=\pi/2}(x, \tau)$ is equal to the fourth component $A_4^E(\vec{x}, \tau)$ of the Euclidean field. The extra factor $e^{-i\theta}$ becomes a factor $-i$ at $\theta = \pi/2$, such that

$$\eta^{\mu\nu} A_\mu(\vec{x}, t) A_\nu(\vec{x}, t) \to \delta^{\mu\nu} A_\mu^E(\vec{x}, \tau) A_\nu^E(\vec{x}, \tau) \tag{3}$$

All this is well-known.

The main idea on which our new approach is based, is the observation that the factor $-i$ in $A_0 \to -iA_4^E$ is part of a **matrix** $A_\mu \to S_\mu{}^\nu A_\nu^E$ which happens to be diagonal for the electromagnetic field

$$\begin{aligned} A_\mu(\vec{x}, t) &\to M_\mu{}^\nu(\theta) A_\nu^\theta(\vec{x}, t_\theta) \\ M_\mu{}^\nu(\theta) &= \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & e^{-i\theta} \end{pmatrix} \end{aligned} \tag{4}$$

Thus, A_μ transforms under Wick rotation as in the theory of induced representations: with an orbital part (the transformation of coordinates) and a spin part (the matrix which acts on the indices of the field). In this sense, the "Wick transformation" is not different from Lorentz transformations, and as we shall see, there are more analogies.

We therefore postulate that Dirac spinors transform under Wick rotations as follows:

$$\psi^\alpha(\vec{x}, t) \to S^\alpha{}_\beta(\theta) \psi_\theta^\beta(\vec{x}, t_\theta) \tag{5}$$

and we must now try to determine this matrix S. However, before going on with the spinorial case, we revert temporarily to the bosonic case to deal with an unusual aspect having to do with complex conjugation. Suppose one were to Wick rotate a complex vector boson field W_μ (for example, the carrier of the weak interactions). Then one would require that

$$\eta^{\mu\nu} W_\mu^* W_\nu \to \delta^{\mu\nu} (W_\mu^E)^* W_\nu^E \tag{6}$$

However, if W_μ^* would transform with the complex conjugate or hermitean conjugate matrix $M_\mu{}^\nu$, the phase factors ($e^{i\theta}$ and $e^{-i\theta}$) would cancel each other. To avoid this,

[3] Fields transform contragrediently to coordinates, so $A^0 \to e^{i\theta} A_\theta^0$. Then $A_0 \to A_0^\theta e^{-i\theta}$ in order that A^0 and A_0 lead to the same $A_4^E = A_E^4$.

we must view complex conjugation as an antilinear operation for Wick rotations (like time reversal) such that

$$W_\mu^*(\vec{x},t) \to M_\mu{}^\nu(\theta) W_\nu^*(\vec{x},t) \tag{7}$$

or

$$W_\mu^*(\vec{x},t) \to M_\nu{}^\mu(\theta) W_\nu^*(\vec{x},t) \tag{8}$$

For bosons, (7) and (8) coincide because $M_\mu{}^\nu$ is diagonal, but for fermions, only one of them will be correct, as we shall discuss.

For $(\psi^\alpha)^\dagger$ we consider two alternatives

$$\psi^\alpha(\vec{x},t)^\dagger \to \psi_\theta^\beta(\vec{x},t_\theta)^\dagger S^\beta{}_\alpha(\theta) \tag{9}$$

or

$$\psi^\alpha(\vec{x},t)^\dagger \to \psi_\theta^\beta(\vec{x},t_\theta)^\dagger S^\alpha{}_\beta(\theta) \tag{10}$$

or in matrix notation $\psi^\dagger \to \psi^\dagger S$ or $\psi^\dagger \to \psi^\dagger S^T$. One might expect that only (10) is correct but not (9) because the matrices S should form a representation of the Wick rotation: $S(\theta_1)S(\theta_2) = S(\theta_1+\theta_2)$. However, since the Wick rotation is an abelian group, $S(\theta_1)$ and $S(\theta_2)$ will commute so that both (9) and (10) satisfy the group compositon law. In fact, (9) is the correct expression.

To fix the matrix S, we now impose three physical requirements. Together they will determine S uniquely. We consider the laws in (5) and (9). The alternative, (10), will be shown later not to be viable.

(1) The Wick rotation does not affect the space coordinates $\vec{x}$, nor should it rotate the space components γ^k of the Dirac matrices. Hence, $S(\theta)$ should depend only on γ^4 and γ^5 but not on γ^1, γ^2 or γ^3

$$[S(\theta), \gamma^k] = 0 \text{ for } k = 1,2,3 \tag{11}$$

We define γ^5 to be hermitean with square one $\gamma^5 \equiv \gamma^1\gamma^2\gamma^3 i\gamma^0$ and the Dirac matrices in Minkowski spacetime satisfy $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$. For future use we define $\gamma^4 \equiv i\gamma^0$.

(2) In order that at $\theta = \pi/2$ the Euclidean action contains the Euclidean Dirac operator

$$\gamma_E^\mu \partial_\mu = \gamma_E^k \partial_k + \gamma_E^4 \partial_4 \; ; \; \{\gamma_E^\mu, \gamma_E^\nu\} = 2\delta^{\mu\nu} \tag{12}$$

we require that $\psi^\dagger i\gamma^0 \to \psi_\theta^\dagger S(\theta) i\gamma^0 = \psi_\theta^\dagger \, M S^{-1}(\theta)$ for some matrix M. Thus

$$S(\theta) i\gamma^0 = M S(\theta)^{-1} \tag{13}$$

If this is the case, then the matrices

$$\begin{aligned}\gamma^k(\theta) &\equiv S^{-1}(\theta)\gamma^k S(\theta) = \gamma^k \\ \gamma^4(\theta) &\equiv S^{-1}(\theta)\gamma^4 S(\theta)\end{aligned} \tag{14}$$

satisfy the Euclidean Clifford algebra at all θ. (At $\theta = \pi/2$ the factor $e^{i\theta}$ from $\partial_t \to e^{i\theta}\partial_{t_\theta}$ becomes a factor i which converts γ^0 into $i\gamma^0 \equiv \gamma^4$.)

(3) In order that at $\theta = \pi/2$ the action has a $SO(4)$ symmetry rather than a $SO(3,1)$ symmetry, we require that M commutes with the $SO(4)$ generators $[\gamma_E^\mu, \gamma_E^\nu]$.

$$[M, [\gamma_E^\mu, \gamma_E^\nu]] = 0 \tag{15}$$

The solution for the Wick rotation matrix $S(\theta)$ is $S(\theta) = e^{\frac{1}{2}\gamma^4\gamma^5\theta}$ where

$$(\gamma^4)^2 = 1, (\gamma^5)^2 = 1, (\gamma^4)^\dagger = \gamma^4, (\gamma^5)^\dagger = \gamma^5. \tag{16}$$

It is clearly independent of γ^k and it is unitary and satisfies

$$S(\theta)i\gamma^0 = i\gamma^0 S(\theta)^{-1} \tag{17}$$

so $M = i\gamma^0$. Explicit evaluation of $\gamma^4(\theta)$ reveals

$$\gamma^4(\theta) = S^{-1}(\theta)\gamma^4 S(\theta) = \gamma^4 \cos\theta + \gamma^5 \sin\theta \tag{18}$$

Hence, $\gamma_E^4 = \gamma^5$, and thus the Euclidean SO(4) generators $[\gamma_E^\mu, \gamma_E^\nu]$ indeed commute with $M = i\gamma^0$. In fact, it is natural to define also a matrix $\gamma^5(\theta)$ by

$$\gamma^5(\theta) \equiv S^{-1}(\theta)\gamma^5 S(\theta) = -\gamma^4 \sin\theta + \gamma^5 \cos\theta \tag{19}$$

so that $\gamma_E^5 = -\gamma^4, \gamma_E^4 = \gamma^5$. The Euclidean action can now be written as

$$\mathcal{L}_E = \psi_E^\dagger(\vec{x}, \tau)\gamma_E^5(\gamma_E^\mu \partial_\mu + m)\psi_E(\vec{x}, \tau) \tag{20}$$

and we see that it is hermitean and SO(4) invariant. The Wick rotation has not removed the matrix $i\gamma^0$; rather it now appears as γ_E^5. Of course $\gamma_E^5 = \gamma_E^1\gamma_E^2\gamma_E^3\gamma_E^4$.

One may ask whether this solution is unique. In order that the Wick rotation forms a one-parameter abelian group, one should be able to write $S(\theta)$ as $\exp\theta N$ for some matrix N which only depends on γ^4, γ^5 and the unit matrix. Pulling $i\gamma^0$ from the right to the left of $S(\theta)$ should result in $i\gamma^0 \exp(-\theta N)$, and this fixes N to be a linear combination of $\gamma^4\gamma^5$ and γ^5.

$$S(\theta) = e^{\theta(\alpha\gamma^4\gamma^5 + \beta\gamma^5)} \tag{21}$$

Finally, SO(4) invariance requires that $i\gamma^0$ anticommutes with $S^{-1}(\theta = \pi/2)\gamma^4 S(\theta = \pi/2)$ (recall that γ^k were inert), which leads to the condition that

$$S(\theta = \pi/2)^2 + S(\theta = \pi/2)^{-2} = 0. \tag{22}$$

whose solution is $S(\theta)$.

The alternative transformation law for $\psi^\dagger$, namely $\psi^\dagger \to \psi^\dagger S^T$, must be rejected, because at $\theta = \pi/2$ the matrices $S(\theta = \pi/2) \equiv S$ must satisfy $S^T\gamma^4 = (\alpha I + \beta\gamma_E^5)S^{-1}$ in order that the Dirac operator be obtained and SO(4) symmetry holds. Using $\gamma_E^5 = S^{-1}\gamma^5 S$ leads to $SS^T = \alpha\gamma^4 + \beta\gamma^5\gamma^4$ which is inconsistent since in general $\alpha\gamma^4 + \beta\gamma^5\gamma^4$ is not a symmetric matrix.

3 Supersymmetry.

Given a supersymmetric theory in Minkowski spacetime with Dirac spinors ψ, vector fields V_μ, scalars A and pseudoscalars B, we can obtain a corresponding supersymmetric theory in Euclidean space by the following substitutions

$$\begin{aligned}
&\psi \to e^{\gamma^4\gamma^5/4}\psi_E, \psi^\dagger \to \psi_E^\dagger e^{\gamma^4\gamma^5/4} \\
&V_\mu = (V_0, V_j) \to (-iV_4^E, V_j^E); d^4x \to -id^4x_E \\
&A \to A_E, B \to iB_E
\end{aligned} \tag{23}$$

The susy parameters are Dirac spinors and must be rotated accordingly:

$$\epsilon \to e^{\gamma^4\gamma^5/4}\epsilon_E, \epsilon^\dagger \to \epsilon_E^\dagger e^{\gamma^4\gamma^5/4} \tag{24}$$

There is a consistency check on our procedure. Consider $\delta\psi = M\epsilon$ for some matrix M. To obtain $\delta\psi^\dagger$ one can either first construct $\delta\psi_E$ and then take the hermitean conjugate, or first take the hermitean conjugate and then perform the Wick rotation.

$$\begin{aligned}
\delta\psi &= M\epsilon \Rightarrow S\delta\psi_E = (M_E)S\epsilon_E \Rightarrow \delta\psi_E^\dagger = \epsilon_E^\dagger S^{-1}(M_E)^\dagger S \\
\delta\psi^\dagger &= \epsilon^\dagger M^\dagger \Rightarrow \delta\psi_E^\dagger S = \epsilon_E^\dagger S(M^\dagger)_E \Rightarrow \delta\psi_E^\dagger = \epsilon_E^\dagger S(M^\dagger)_E S^{-1}
\end{aligned} \tag{25}$$

Hence, the matrix M in $\delta\psi = M\epsilon$ must satisfy

$$S^{-1}(M_E)^\dagger S = S(M^\dagger)_E S^{-1} \tag{26}$$

where M_E and $(M^\dagger)_E$ are obtained from M and $M^\dagger$ by using the substitutions in (23). In particular, V_μ^E, A^E and B^E are to be considered real fields if V_μ, A and B were real. The $N = 2$ model of Zumino satisfies this consistency check. The origin of the factor i in $B \to iB_E$ becomes clear if one takes as a model for a pseudoscalar

$$B = \epsilon^{\mu\nu\rho\sigma}\partial_\mu\varphi^1\partial_\nu\varphi^2\partial_\rho\varphi^3\partial_\sigma\varphi^4 \tag{27}$$

where φ^i are real scalars. Clearly, B picks up a factor i under the Wick rotation, since one of the derivatives is $\partial/\partial t$.

The action one obtains in this way contains a matrix γ_E^5 as we have seen, but for massless fields its presence may be overlooked because one might use $i\gamma_E^5\gamma_E^\mu$ as Dirac matrices. In this way one recovers the model of Zumino.

4 Majorana and Weyl spinors.

For Majorana spinors, we relax the reality condition on spinors, like Nicolai. Applying our transformation to the fields ψ and $\psi^T C$ in the Minkowski action, we transform them as

$$\psi \to S\psi, (\psi^T C) \to \psi^T S^T C = \psi^T(S^T C S)S^{-1} \tag{28}$$

The matrix $C_E \equiv S^T C S$ has the same symmetry properties as C, hence for free spinors we get essentially the same action as Nicolai (with C_E instead of C)

$$\mathcal{L}_E = \psi_E^T C_E (\gamma_E^\mu \partial_\mu + m) \psi_E \tag{29}$$

However, all interactions in Euclidean space automatically follow from the Minkowski theory; one must just insert the matrices S and work out consequences. We have constructed some $N = 1$ examples which will be published elsewhere.

For Weyl spinors $\psi_L = \frac{1}{2}(1+\gamma^5)\psi$ and $\psi_L^\dagger = \frac{1}{2}\psi^\dagger(1+\gamma_5)$ in Minkowski spacetime, are related by complex conjugation. Of course, in $d = 4$ Minkowski spacetime one can rewrite a Majorana spinor as a Weyl spinor. Hence, for consistency we relax the relation between ψ_L and $\psi_L^\dagger$ before we begin the Wick rotation. For clarity we write $\psi^\dagger = \chi^\dagger$. Then we start from

$$\mathcal{L}(\text{Weyl}) = \chi^\dagger \left(\frac{1+\gamma_5}{2}\right) \gamma^4 \partial\!\!\!/ \left(\frac{1+\gamma_5}{2}\right) \psi \tag{30}$$

and we rotate as before: $\psi \to S\psi_E, \chi^\dagger \to \chi_E^\dagger S$. Then

$$\psi_L \to S\frac{(1+\gamma_E^5)}{2}\psi_E, \chi_L^\dagger \gamma^4 \to \chi_E^\dagger \gamma^4 \left(\frac{1-\gamma_E^5}{2}\right) S^{-1} \tag{31}$$

Hence, using $\gamma^4 = -\gamma_E^5$, we find

$$\mathcal{L}(\text{Weyl}) \to \chi_E^\dagger \left(\frac{1-\gamma_E^5}{2}\right) \gamma^4 \partial\!\!\!/^E \left(\frac{1+\gamma_E^5}{2}\right) \psi = \chi_R^{E\dagger} \partial\!\!\!/^E \psi_L^E \tag{32}$$

As expected, the $\bar{\chi}_L \partial\!\!\!/ \psi_L$ pairing has become a $(\chi_R^E)^\dagger \partial\!\!\!/^E \psi_L^E$ pairing, as required by SO(4) invariance.

5 Discussion.

An interpretation of our results in a five-dimensional spacetime with signature $(4,1)$ is possible. The matrix $\gamma^4\gamma^5$ in $S(\theta)$ is a Lorentz generator in this space. Also in the canonical work of OS where all fields commute for all points in space, there are 5 dimensional echoes: one seems to be working at equal time $t = 0$ in a 4-dimensional space $(\vec{x}, \tau)$. Additional evidence are the normalization factors $(k^2 + m^2)^{-1/2}$ where $k^2 = \vec{k}^2 + k_4^2$ which appear in the second-quantized fermion fields of OS. They are clearly the d=4 generalization of the usual d=3 factors $(2\omega)^{-1/2}$ where $\omega^2 = \vec{k}^2 + m^2$. Further the mode operators in their work depend on Euclidean four-momenta. Also in Schwinger's work one finds 5-dimensional hints. He introduces for each field $A(x)$ another field $B(x)$ satisfying $[A(x), B(y)] = \delta^4(x - y)$, which can again be viewed as equal-time 5 dimensional canonical commutation rules.

If one views the five-dimensional space as flat, with a flat vielbein ${e_A}^M$, then the complex general coordinate transformation $t \to e^{-i\theta} t_\theta$ acts on the index M of the vielbein, and one needs a compensating Lorentz transformation to keep ${e_A}^M$ flat; this is just our 5-dimensional Lorentz rotation. We are aware of many loose ends which have to be better understood.

Coming back to the relation between Schwinger's approach and that of OS, from our point of view, Schwinger's results are due to rotation with our matrix S, (including the matrix γ_E^5), while OS results can be viewed simply by defining $\psi^\dagger \gamma_E^5$ as a new complex spinor $\chi^\dagger$. Of course, the OS approach is really based on canonical quantization, and in Euclidean space one needs twice as many creation and annihilation operators. However, from a path integral point of view, there is less difference. Grassmann integration over ψ and $\psi^\dagger$ has as many degrees of freedom as integration over $\chi^\dagger$ and ψ. The Grassmann integral does not see reality properties, being based only on the rule $\int d\psi \psi = 1, \int d\psi = 0$. We believe that with our results in a path integral context, the Wick rotation of spinors becomes as clear as for bosons.